Statistics

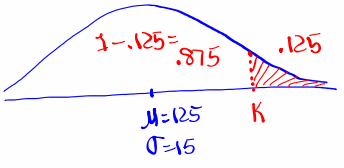
Lecture 11

Feb 19-8:47 AM

Consider a population normally dist. with the mean of 125 and standard deviation of 15

$N(125, 15)$

1) Find K such that $P(x > K) = .125$ (Right Area)



K = invNorm(.875, 125, 15)

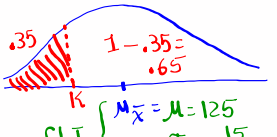
= 142.255

≈ 142

2) Find K such that $P(\bar{x} < K) = .35$ (Left Area) for randomly selected groups of 6.

K = invNorm(.35, 125, 15/√6)

= 122.640 ≈ 123



CLT $\mu_{\bar{x}} = \mu = 125$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{6}}$

Drawing, Labeling, Shading, and full TI Command required.

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SG 22-23

Population ⟷ Parameter

Sample ⟷ Statistic

Estimating Parameters

we can estimate

1) Population Proportion p

2) Population Mean μ

3) Population Standard deviation σ

To do the estimation, we start with

Point-estimate (Best guess)

Use Sample Proportion $\hat{p}$ p-hat to estimate Population Proportion p.

Use Sample Mean $\bar{x}$ x-bar to estimate Population Mean μ.

Use Sample Standard deviation s to estimate Population Standard deviation σ.

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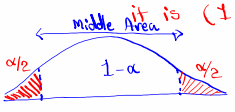
When estimating a parameter, the answer is range of values which is called

Confidence Interval

Every confidence interval comes with

Confidence level.

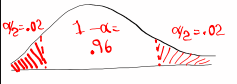
↳ is the middle area of the Prob. dist. curve and it is $(1-\alpha)\cdot 100\%$.



Alpha $0 < \alpha < 1$ Significance level

For example

$\alpha = .04$, $\alpha/2 = .02$, $1-\alpha = .96$



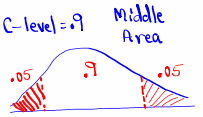
Conf. level = 90% → C-level = .9

$1 - .9 = .1 \leftarrow \alpha$

$\alpha/2 = .05$

Right Tail Area

Left Tail Area



α → Significance level

$\alpha/2$ → Area of each tail

$1-\alpha$ → Middle Area

$(1-\alpha)\cdot 100\%$ → C-level

If α not given → use .05

If C-level not given → use 95%

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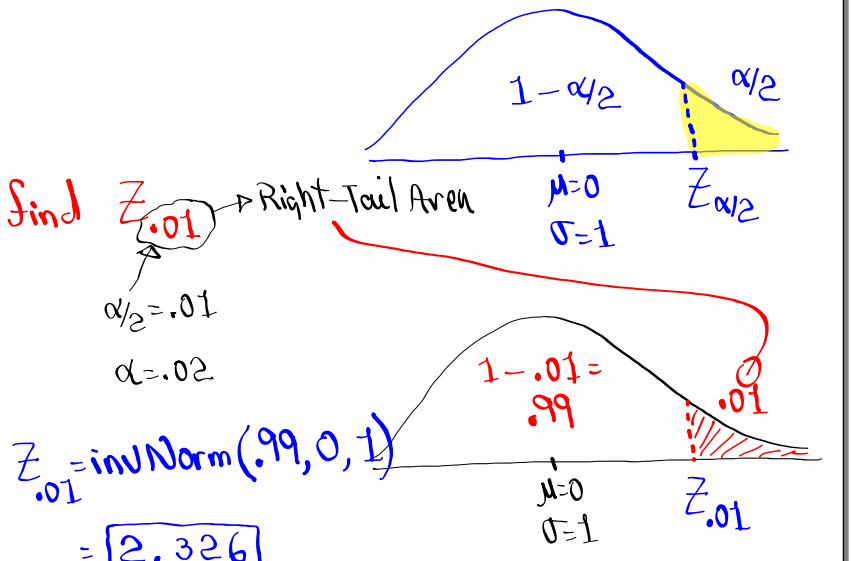
$z_{\alpha/2}$ is a value with Standard-Normal Prob. dist with right-Tail Area of $\alpha/2$.



Find $z_{.01}$ → Right-Tail Area

$\alpha/2 = .01$

$\alpha = .02$

$z_{.01} = \text{invNorm}(.99, 0, 1)$

$= \boxed{2.326}$

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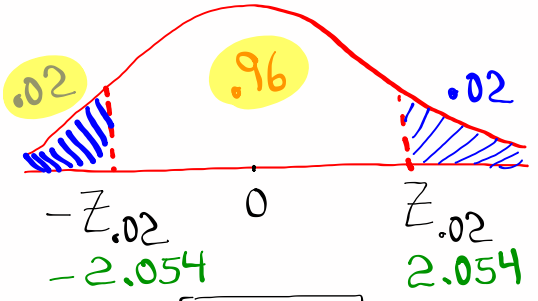
Find $\pm z_{\alpha/2}$ for 96% C-level.

middle area .96

$1 - .96 = .04 \leftarrow \alpha$

$.04/2 = .02 \leftarrow \alpha/2$



$z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

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Find $Z_{\alpha/2}$ when α not given.

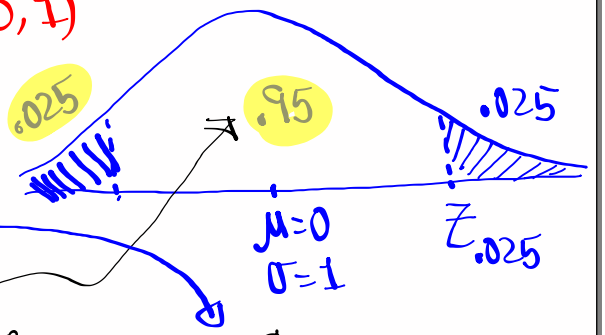
use $\alpha = .05$

$\alpha/2 = .05/2 = .025$

C-level $1-\alpha = .95 = 95\%$

$Z_{.025} = \text{invNorm}(.975, 0, 1)$

≈ 1.960



Recall Usual Range

95% Range $\mu \pm 2\sigma$

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T- Dist.

1) Bell-shape, Symmetric, Total area = 1

2) $\mu = 0$, σ unknown

3) It comes with degrees of freedom df

$t_{\alpha/2}$ is like $Z_{\alpha/2}$

we use invT to find it.

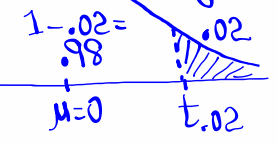
Find $t_{.02}$ with df = 15.

$t_{.02} = \text{invT}(.98, 15)$

Left Area, df

2nd | VARS

$= 2.249$



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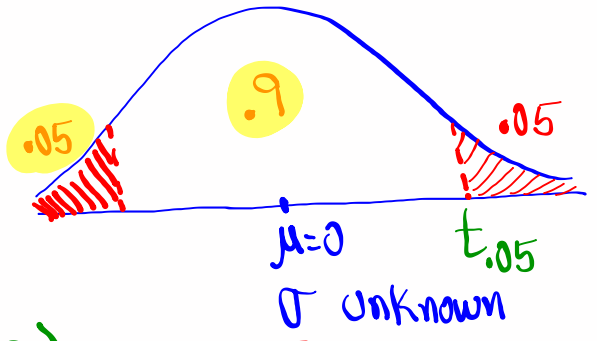
Find $t_{\alpha/2}$ for 90% C-level with df=12.

Middle area = .9

$1 - .9 = .1 \leftarrow \alpha$

$.1 \div 2 = .05 \leftarrow \alpha/2$



$t_{.05} = \text{invT}(.95, 12) \approx 1.782$

(.95: Left Area; 12: df)

Apr 23-7:42 PM

Estimating Population Proportion P:

$\ldots < P < \ldots$

$\hat{P} - E < P < \hat{P} + E$

$\hat{P}$: P-hat, Sample Proportion, Point-estimate

E: Margin of error

$\hat{p} = \frac{x}{n}$ — x: # of favorable responses; n: Sample Size

$\hat{q} = 1 - \hat{p}$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$

$Z_{\alpha/2}$: Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

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I surveyed 100 students, and 80 of them had iPhone.

$n = 100$, $x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$ $\quad \hat{q} = 1 - \hat{p} = 1 - .8 = .2$

$\hat{p} - E < p < \hat{p} + E$

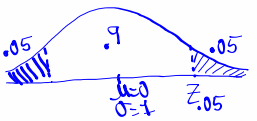
$.8 - E < p < .8 + E$

Suppose C-level = 90%.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\frac{(.8)(.2)}{100}}$

$E = \boxed{.07}$



$z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

$.8 - .07 < p < .8 + .07$

$\boxed{.73 < p < .87}$

We are 90% confident that between 73% and 87% of all students have iPhone.

Using TI:

STAT → TESTS ↓ 1-PropZInt

$x = 80$

$n = 100$

C-level: .9

Calculate

$.73 < p < .87$

$E = \frac{.87 - .73}{2} = \boxed{.07}$

$\hat{p} = \frac{.87 + .73}{2} = \boxed{.8}$

Apr 23-8:02 PM

I surveyed 240 students, and 65% of them were fan of online classes.

$n = 240$

$\hat{p} = .65$

$x = n\hat{p} = 240(.65) = 156$

if decimal → Round-up

Find Conf. interval for the prop. of all students that are fan of online classes.

NO C-level

⇒ use .95

STAT TESTS

1-PropZInt

$x = 156$

$n = 240$

C-level: .95

Calculate

$< p <$

$.59 < p < .71$

We are 95% confident that between 59% & 71% of all students are in favor of online classes.

$E = \frac{.71 - .59}{2} = \boxed{.06}$

$\hat{p} = \frac{.71 + .59}{2} = \boxed{.65}$

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I surveyed 175 students and 8% of them were Smokers.

$n = 175$ $\quad \hat{p} = .08$ $\quad x = n\hat{p} = 175(.08) = 14$

Find 99% Conf. interval for the prop. of all Students that are Smokers.

C-level: .99

1-PropZInt

$x = 14$

$n = 175$

C-level: .99

$.03 < p < .13$

We are 99% Confident that between 3% & 13% of all Students are Smokers.

$E = \frac{.13 - .03}{2} = .05$

$\hat{p} = \frac{.13 + .03}{2} = .08$

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Estimating Population Mean μ:

$\cdots < \mu < \cdots$

$\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean, Point-estimate

E: Margin of error

Case I: σ Known

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

STAT TESTS

TI: Z Interval

inpt: STATS

$E = \frac{-}{2}$, $\bar{x} = \frac{+}{2}$

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Given $n=40$, $\bar{x}=125$, $\sigma=20$, C-level: .98

Find Conf. interval for pop. mean.

σ is Known

Use ZInterval

inpt: Stats

$\sigma=20$

$\bar{x}=125$

$n=40$

C-level: .98

Calculate

$118 < \mu < 132$

We are 98% Confident that pop. mean is between 118 and 132.

$E = \frac{132-118}{2} = 7$

$\bar{x} = \frac{132+118}{2} = 125$

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I surveyed 32 students, their mean age was 28.5 Yrs. $n=32$, $\bar{x}=28.5$

It is known that standard dev. of ages of all students is 7.2 Yrs. $\sigma=7.2$

Find 90% Conf. interval for the mean age of all students. $\_\_ < \mu < \_\_$

C-level: .9

σ Known → ZInterval

inpt: Stats

$\sigma=7.2$

$\bar{x}=28.5$

$n=32$

C-level: .9

Calculate

Round to 1-decimal →

$26.4 < \mu < 30.6$

$E = \frac{30.6-26.4}{2} = 2.1$

$\bar{x} = \frac{30.6+26.4}{2} = 28.5$

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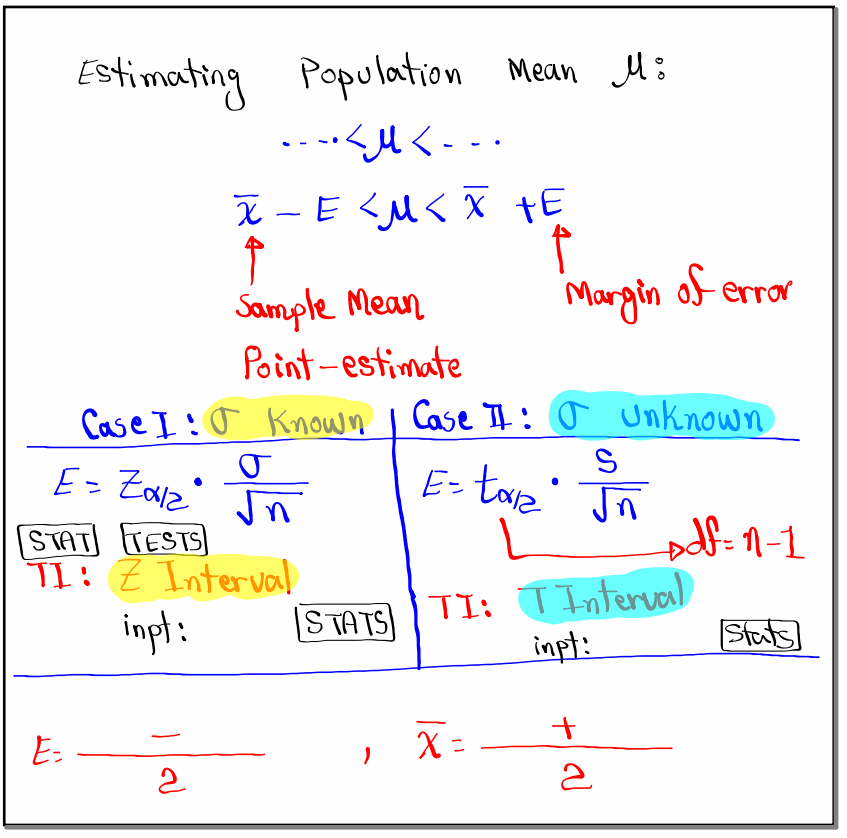

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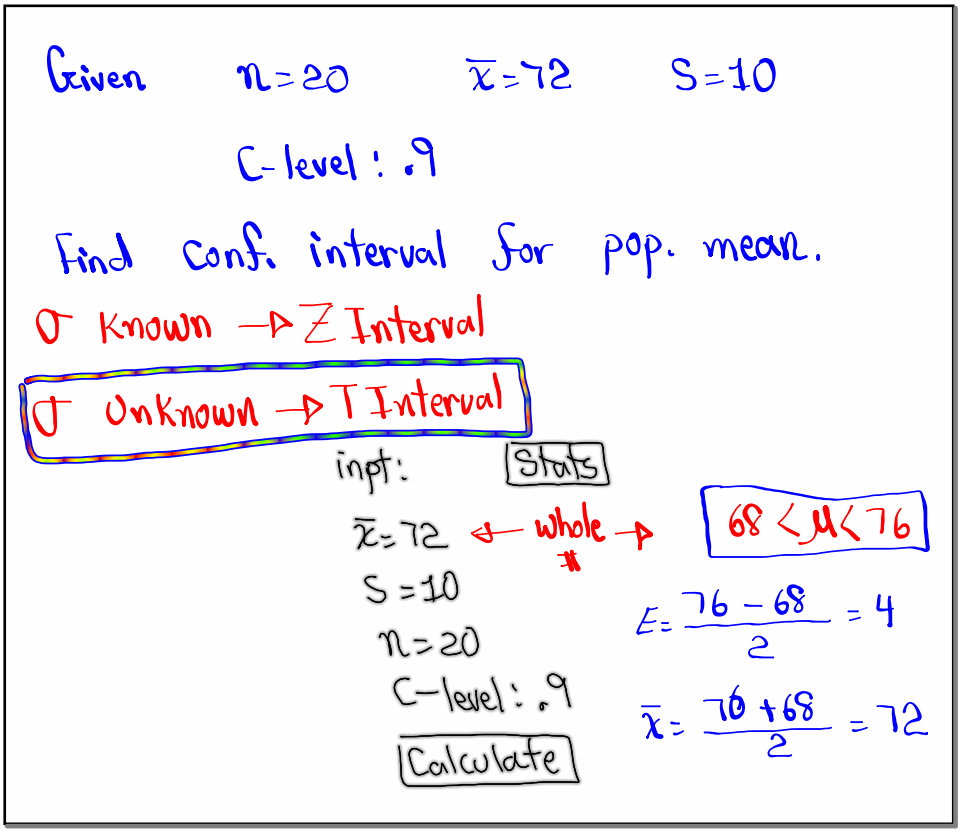

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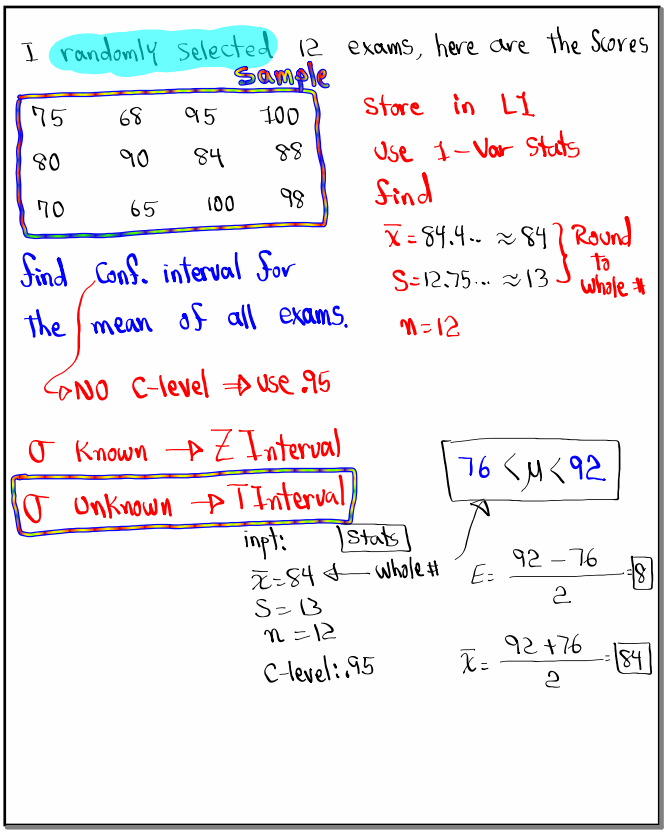
I randomly selected 12 exams, here are the scores
Sample
75 68 95 100
80 90 84 88
70 65 100 98
Store in L1
Use 1-Var Stats
Find
$\bar{x}$ = 84.4... ≈ 84
S = 12.75... ≈ 13
Round to whole #
n = 12
Find Conf. interval for the mean of all exams.
NO C-level ⇒ use .95
σ Known → Z Interval
σ Unknown → T Interval
inpt: Stats
$\bar{x}$ = 84 ← whole #
S = 13
n = 12
C-level: .95
76 < μ < 92
E = (92 − 76)/2 = 8
$\bar{x}$ = (92 + 76)/2 = 84

Apr 23-9:04 PM